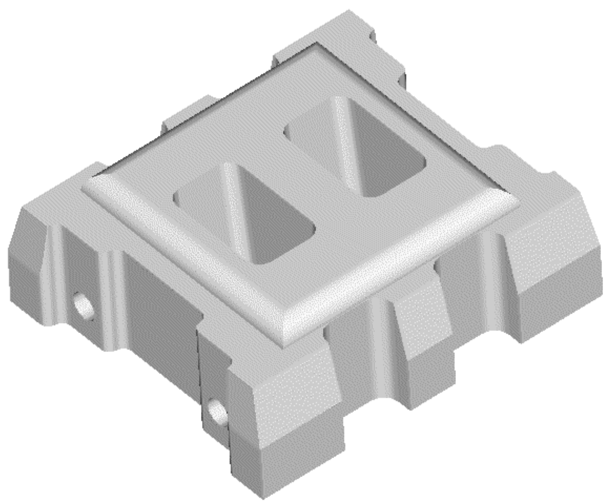
ArmorFlex Design Manual

ABRIDGED VERSION 2002

Design Manual for ArmorFlex® Articulating Concrete Blocks



ARMORTEC
Erosion Control Solutions

1. INTRODUCTION

This document is an abridged version of the full ArmorFlex® Design Manual, available from Armortec. This manual was developed specifically to supplement the ArmorFlex Design Package and includes only the supporting documentation for the design equations used by the software application. Subsequent sections present development of the design equations, a design example, and references.

Subjects that have been omitted from this manual, which are included in the full version, include open channel hydraulics, laboratory testing, installation, and other special topics. The user is encouraged to contact Armortec for a full version of the ArmorFlex® Design Manual for discussion of subjects beyond the scope of this document.

2. DESIGN EQUATIONS AND EXAMPLE

2.1 Hydraulic Stability of the ArmorFlex® Systems

Due to the mode of failure exhibited by articulating concrete block revetment systems, the most vulnerable component of an ArmorFlex® revetment system is an individual block located at the toe of a channel side slope. The approach used to determine the stability of the block is similar to the approach used by Simons and Senturk for riprap sizing (ref.6).

The forces acting on a block placed on the side slope of a channel are shown in **Figure 2.1**. The following list contains a description of the major variables:

λ	=	angle (in degrees) of the channel bed
θ	=	angle (in degrees) of the side slope
β	=	angle (in degrees) between the weight vector and the resultant force vector in the plane of the side slope
δ	=	angle (in degrees) between the drag vector and the resultant force vector in the plane of the side slope
F_D	=	drag force acting on the block (lbs)
F_L	=	lift force acting on the block (lbs)
$\ell_1, \ell_2, \ell_3, \ell_4$	=	moment arms for respective forces (ft)
W_s	=	buoyant (submerged) weight of the block (lbs)

As can be seen in Figure 2.1, the moments of overturning will act along the resultant force, R, about the point O. Point O in this case is the downstream and downslope corner of the block. If the block is in equilibrium, the sum of moments about point O along R must equal zero. The summation of the moments about O results in:

$$\sum M_o = 0 = \ell_2 W_s \cos\theta - \ell_1 W_s \sin\theta \cos\beta - \ell_3 F_D \cos\delta + \ell_4 F_L \qquad (2.1)$$

The factor of safety, SF, is the ratio of the resisting moments to the overturning moments. For the ArmorFlex® block the factor of safety is:

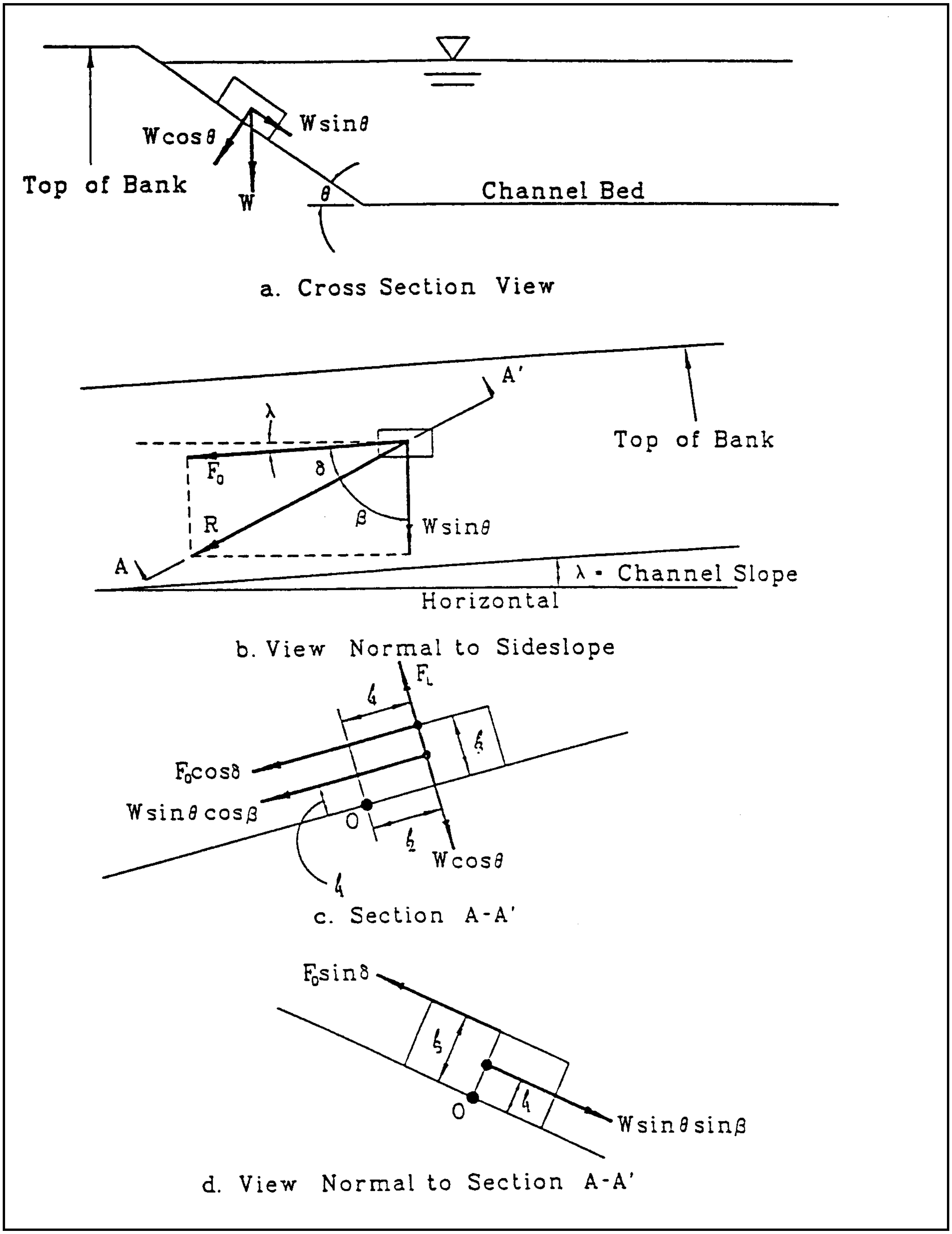


Figure 2.1. Force diagrams for an ArmorFlex® block on a channel side slope.

$$SF = \frac{M_{resisting}}{M_{overturning}} = \frac{\ell_2 W_s \cos\theta}{\ell_1 W_s \sin\theta \cos\beta + \ell_3 F_D \cos\delta + \ell_4 F_L} \tag{2.2}$$

by dividing both the numerator and the denominator by $\ell_1 W_S$:

$$SF = \frac{\frac{\ell_2}{\ell_1}\cos\theta}{\sin\theta\cos\beta + \frac{\ell_2}{\ell_1}\eta'} \tag{2.3}$$

with η' being:

$$\eta' = \frac{\ell_3 F_D \cos\delta}{\ell_2 W_s} + \frac{\ell_4 F_L}{\ell_2 W_s} \tag{2.4}$$

For motion along R, the moments of the forces perpendicular to R (Figure 2.1d) must balance. This result is:

$$\ell_3 F_D \sin\delta = \ell_1 W_s \sin\theta \sin\beta \tag{2.5}$$

solving Equation 2.5 for $\sin\beta$:

$$\sin\beta = \frac{\ell_3 F_D \sin\delta}{\ell_1 W_s \sin\theta} \tag{2.6}$$

from the definition of δ in Figure 2.1 and basic geometric relations

$$\cos\delta = \cos(90 - \lambda - \beta) = \sin(\lambda + \beta) \tag{2.7}$$

$$\sin\delta = \sin(90 - \lambda - \beta) = \cos(\lambda + \beta) \tag{2.8}$$

$$\sin\delta = \cos\lambda\cos\beta - \sin\lambda\sin\beta \tag{2.9}$$

substituting Equation 2.9 in Equation 2.6:

$$\sin\beta = \frac{\ell_3 F_D \cos\lambda\cos\beta - \ell_3 F_D \sin\lambda\sin\beta}{\ell_1 W_s \sin\theta} \tag{2.10}$$

dividing both sides by $\sin\beta$ and rearranging terms:

$$\frac{\ell_3 F_D \cos\lambda\cos\beta}{\sin\beta} = \ell_1 W_s \sin\theta + \frac{\ell_3 F_D \sin\lambda\sin\beta}{\sin\beta} \tag{2.11}$$

substituting $\tan\beta$ for $\sin\beta/\cos\beta$ and solving for $\tan\beta$:

$$\tan\beta = \frac{\ell_3 F_D \cos\lambda}{\ell_1 W_s \sin\theta + \ell_3 F_D \sin\lambda} \tag{2.12}$$

dividing both the numerator and denominator by $\ell_3 F_D$:

$$\tan\beta = \frac{\cos\lambda}{\frac{\ell_1 W_s}{\ell_3 F_D}\sin\theta + \sin\lambda} \tag{2.13}$$

for flow over a plane flat bed, where $\lambda = 0$ and $\theta = 0$:

$$\eta = \frac{\ell_3 F_D}{\ell_2 W_s} + \frac{\ell_4 F_L}{\ell_2 W_s} \tag{2.14}$$

for this situation Equation 2.3 becomes:

$$SF = \frac{1}{\eta} \tag{2.15}$$

when the safety factor is unity (incipient conditions at the threshold of motion) η becomes 1, for other conditions:

$$\eta = \frac{\tau_O}{\tau_C} \tag{2.16}$$

where: τ_O = maximum shear stress on channel (lbs/ft^2)
τ_C = critical shear stress of lining (lbs/ft^2)

relating η' to η:

$$\frac{\eta'}{\eta} = \frac{\frac{\ell_3 F_D}{\ell_2 W_s}\cos\delta + \frac{\ell_4 F_L}{\ell_2 W_s}}{\frac{\ell_3 F_D}{\ell_2 W_s} + \frac{\ell_4 F_L}{\ell_2 W_s}} \tag{2.17}$$

for simplification define the ratio M/N:

$$M/N = \frac{\ell_4 F_L}{\ell_3 F_D} \tag{2.18}$$

substituting Equations 2.7 and 2.18 into Equation 2.17 and reducing:

$$\eta' = \eta\left(\frac{M/N + \sin(\lambda+\beta)}{M/N + 1}\right) \tag{2.19}$$

solving Equation 2.14 for $\frac{\ell_3 F_D}{\ell_2 W_s}$ and substituting Equation 2.18:

$$\frac{\ell_3 F_D}{\ell_2 W_s} = \frac{\eta}{\left(\frac{\ell_4 F_L}{\ell_3 F_D} + 1\right)} = \frac{\eta}{M/N + 1} \quad (2.20)$$

rearranging the term $\frac{\ell_1 W_s}{\ell_3 F_D}$ from Equation 2.13 and substituting Equation 2.20:

$$\frac{\ell_1 W_s}{\ell_3 F_D} = \frac{\ell_2 W_s}{\ell_3 F_D}\frac{\ell_1}{\ell_2} = \frac{M/N + 1}{\eta}\frac{\ell_1}{\ell_2} \quad (2.21)$$

substituting Equation 2.21 into Equation 2.13:

$$\tan\beta = \frac{\cos\lambda}{\frac{M/N + 1}{\eta}\frac{\ell_1}{\ell_2}\sin\theta + \sin\lambda} \quad (2.22)$$

Table 2.1 contains a summary of the equations used in the factor of safety procedure for the ArmorFlex® block systems.

In order to solve for the factor of safety equations the following need to be known; ℓ_1, ℓ_2, ℓ_3, ℓ_4, λ, θ, τ_C, and τ_O. **Table 2.3** contains values for the variables associated with the different classes of ArmorFlex® concrete blocks. Due to the difficulty of determining the lift force, F_L, it is set equal to the drag force, F_D, as a conservative estimate; therefore, $\frac{F_L}{F_D} = 1$.

Table 2.1 Factor of Safety Equations.	
$$SF = \frac{\frac{\ell_2}{\ell_1}\cos\theta}{\sin\theta\cos\beta + \frac{\ell_2}{\ell_1}\eta'}$$	(2.23)
$$\beta = \tan^{-1}\left[\frac{\cos\lambda}{\frac{M/N+1}{\eta}\frac{\ell_1}{\ell_2}\sin\theta + \sin\lambda}\right]$$	(2.24)
$$\eta' = \eta\left(\frac{M/N + \sin(\lambda+\beta)}{M/N+1}\right)$$	(2.25)
$$\eta = \frac{\tau_O}{\tau_C}$$	(2.26)
$$M/N = \frac{\ell_4}{\ell_3}$$	(2.27)

2.2 Stability Analysis for Projecting Blocks

The previous section assumed that the ArmorFlex® blocks were placed to achieve a flush, smooth surface with no projecting blocks. In reality, achieving such an installation requires precise subgrade preparation. The following section describes the theory that takes into account the additional forces that affect a projecting block.

The factor of safety analysis for a projecting block is the same as in the previous section, with the addition of an extra drag force term, F_D’, which accounts for the impact of the flow against the projecting face. The resulting equation is:

$$SF = \frac{M_{resisting}}{M_{overturning}} = \frac{\ell_2 W_s \cos\theta}{\ell_1 W_s \sin\theta\cos\beta + \ell_3 F_D \cos\delta + \ell_3 F_D'\cos\delta + \ell_4 F_L + \ell_4 F_L'} \quad (2.28)$$

dividing the numerator and denominator by $\ell_1 W_s$:

$$SF = \frac{\frac{\ell_2}{\ell_1}\cos\theta}{\sin\theta\cos\beta + \frac{\ell_2}{\ell_1}\eta' + \frac{\ell_3 F_D'}{\ell_1 W_s}\cos\delta + \frac{\ell_4 F_L'}{\ell_1 W_s}} \quad (2.29)$$

with η' the same as in the previous section:

$$\eta' = \frac{\ell_3 F_D \cos\delta}{\ell_2 W_s} + \frac{\ell_4 F_L}{\ell_2 W_s} \tag{2.30}$$

substituting Equation 2.7 into Equation 2.28:

$$SF = \frac{\frac{\ell_2}{\ell_1}\cos\theta}{\sin\theta\cos\beta + \frac{\ell_2}{\ell_1}\eta' + \frac{\ell_3 F_D'}{\ell_1 W_s}\sin(\lambda+\beta) + \frac{\ell_4 F_L'}{\ell_1 W_s}} \tag{2.31}$$

The additional drag force, F_D', caused by the projecting block is determined using the impulse-momentum principal. The drag force equation is:

$$F_D' = \rho C_D A_N V^2 \tag{2.32}$$

where:

F_D'	=	drag force (lbs.) caused by projecting block
ρ	=	density of water (slugs/ft^3)
C_D	=	coefficient of drag
A_N	=	area of projecting face normal to the flow (ft^2)
V	=	flow velocity (ft/s)

For the case of a projecting block, the area normal to the flow, A_N, is equal to the area created by the projection. A drag coefficient, C_D, of 0.5 has been found to adequately model a projecting block. Substituting the area and the drag coefficient into Equation 2.31 the drag force becomes:

$$F_D' = 0.5\rho(\Delta z)wV^2 \tag{2.33}$$

where:

Δz	=	height of projecting surface normal to the flow (ft)
w	=	width of projecting surface normal to the flow (ft)

The additional lift force due to the projecting block, F_L', is assumed equal to the additional drag force, F_D', as a conservative estimate. Block placement precision in laboratory testing is generally considered to be in the range of +/- 0.25 inches. In practice, the designer specifies a project-specific block placement tolerance, where a minimum value of 0.5 inches is recommended unless specific circumstances allow for more precision in block installation. **Table 2.2** contains a summary of the equations used to determine the factor of safety for a projecting ArmorFlex® block.

{tc " " \l 2}Table 2.2 Factor of Safety Equations for Projecting Block.	
$SF = \dfrac{\dfrac{\ell_2}{\ell_1}\cos\theta}{\sin\theta\cos\beta + \dfrac{\ell_2}{\ell_1}\eta' + \dfrac{\ell_3 F_D'}{\ell_1 W_s}\sin(\lambda+\beta) + \dfrac{\ell_4 F_L'}{\ell_1 W_s}}$	(2.34)
$\beta = \tan^{-1}\left[\dfrac{\cos\lambda}{\dfrac{M/N+1}{\eta}\dfrac{\ell_1}{\ell_2}\sin\theta + \sin\lambda}\right]$	(2.35)
$F_D' = 0.5\rho\Delta z w V^2$	(2.36)
$\eta = \dfrac{\tau_O}{\tau_C}$	(2.37)
$\eta' = \eta\left(\dfrac{M/N + \sin(\lambda+\beta)}{M/N+1}\right)$	(2.38)
$M/N = \dfrac{\ell_4}{\ell_3}$	(2.39)

In order to solve the factor of safety equations, the following need to be known; ℓ_1, ℓ_2, ℓ_3, ℓ_4, ρ, Δz, w, V, λ, θ, τ_C, and τ_O. **Table 2.3** contains the values for the variables associated with the different classes of ArmorFlex® concrete blocks. Data used in the development of this information were measured or extrapolated from the testing programs shown below. Tests are divided into two categories: with and without the use of a drainage layer. Because it is known that drainage layers increase block performance, blocks that have been tested with a drainage layer should be designed and constructed with a similar drainage system. The blocks that are designed from test data that incorporated a drainage layer are all the open-cell tapered blocks (-T).

1. Blocks tested without the use of a drainage layer:
   - ArmorFlex® Class 30-S (FHWA RD-88-181, November 1988)
   - ArmorFlex® Class 40-L (Ayres Associates, Proj. No. 34-0705, July 2000)
   - ArmorFlex® Class 45-L (Ayres Associates, Proj. No. 34-0705, July 2000)

2. Blocks tested with a drainage layer (synthetic or granular) between the geotextile and the revetment:
   - ArmorFlex® Class 40-T (Ayres Associates, Proj. No. 34-0705, May 2000)

Table 2.3. Factor of Safety Equation Variables.					
Block Class	Submerged Weight (Lbs)	ℓ_1 (ft)	ℓ_2 & ℓ_4 (ft)	ℓ_3 (ft)	τ_c @ 0 degrees (psf)
30-S	19.80	0.198	0.726	0.317	14.40
50-S	28.60	0.250	0.726	0.400	19.00
45-S	24.50	0.198	0.726	0.317	17.90
55-S	33.30	0.250	0.726	0.400	22.10
40	37.30	0.198	0.971	0.317	22.40
50	47.80	0.250	0.971	0.400	26.60
60	60.60	0.313	0.971	0.500	31.00
70	75.30	0.375	0.971	0.600	35.50
45	45.50	0.198	0.971	0.317	27.30
55	58.30	0.250	0.971	0.400	32.80
75	74.60	0.313	0.971	0.500	38.20
85	91.00	0.375	0.971	0.600	43.00
40-L	46.80	0.198	1.222	0.317	25.80
50-L	60.30	0.250	1.222	0.400	30.50
60-L	74.90	0.313	1.222	0.500	35.60
70-L	90.00	0.375	1.222	0.600	40.80
45-L	56.20	0.198	1.222	0.317	31.00
55-L	72.30	0.250	1.222	0.400	37.20
75-L	90.00	0.313	1.222	0.500	43.20
85-L	108.70	0.375	1.222	0.600	48.70
40-T	35.50	0.198	0.971	0.317	31.80
50-T	44.80	0.250	0.971	0.400	36.90
60-T	56.00	0.313	0.971	0.500	42.10
70-T	67.20	0.375	0.971	0.600	46.50

NOTE: Moment arms and critical shear stresses assume block orientation of the block with the long axis parallel to flow.

2.3 Design Example

Using a given channel design, assess the hydraulic stability of the ArmorFlex® 45-L block assuming a vertical projection of ½ inch, with unvegetated (e.g., newly installed) conditions. In a field application when considering installation variables, this type of offset can occur. The following example will illustrate the potential instability caused by additional forces affecting a projecting block. The previously described theory will be utilized to determine the factor of safety.

Given: 100-year design discharge: Q =500 ft^3/s
Channel slope: S_0 = 0.05 ft/ft
Channel side slope Z = 3H:1V
Channel bend radius R_c = 80 ft
Bottom width b = 10 ft
Required factor of safety: 1.50

Step 1

Determine the normal depth, y, for a bottom width of 10 ft; assume n = 0.032:
From the nomograph of **Figure 2.2**,

for Bottom width b = 10 ft, y = 2.19 ft

Step 2

Determine velocity and bed shear stress (including bend correction):

a. Velocity: V = Q/A = Q/y(b+Zy) = (500 ft^3/s) / (36.28 ft^2) = 13.78 ft/s

b. Maximum shear stress: $\tau_0 = K_b \gamma y S_0$

From **Figure 2.3**, with R_c/b = 80/10 = 8.0, determine K_b = 1.2

τ_0 = (1.2) x (62.4 lb/ft^3) x (2.19ft) x (0.05 ft/ft) = 8.20 lb/ft^2

Table 2.2 outlines the procedure for determining the factor of safety for a block, which exhibits a specified projection height. From steps one and two above,

y = 2.19 ft
V = 13.78 ft/s
τ_0 = 8.20 lb/ft^2

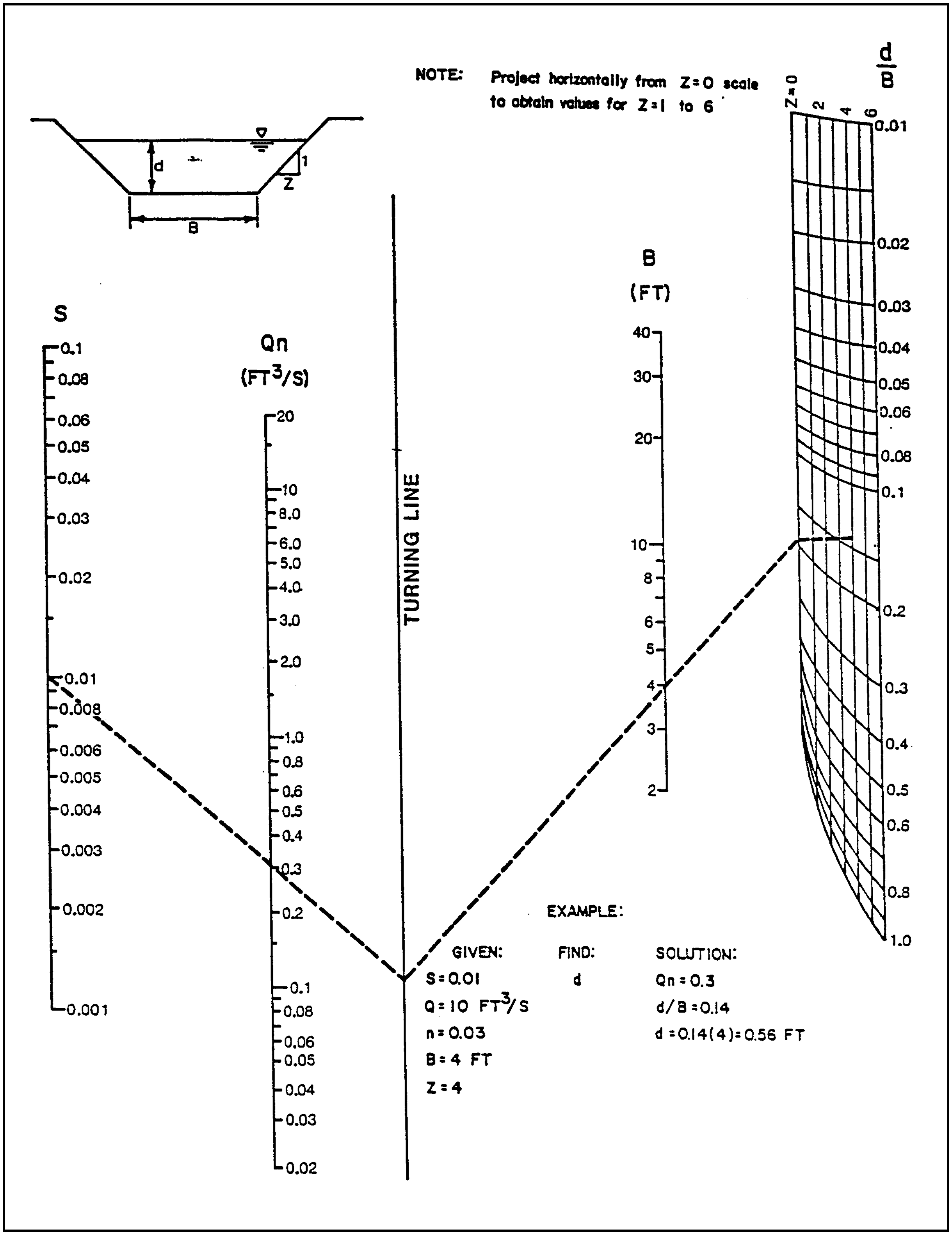


Figure 2.2. Nomographic solution of Manning's equation for trapezoidal channels.

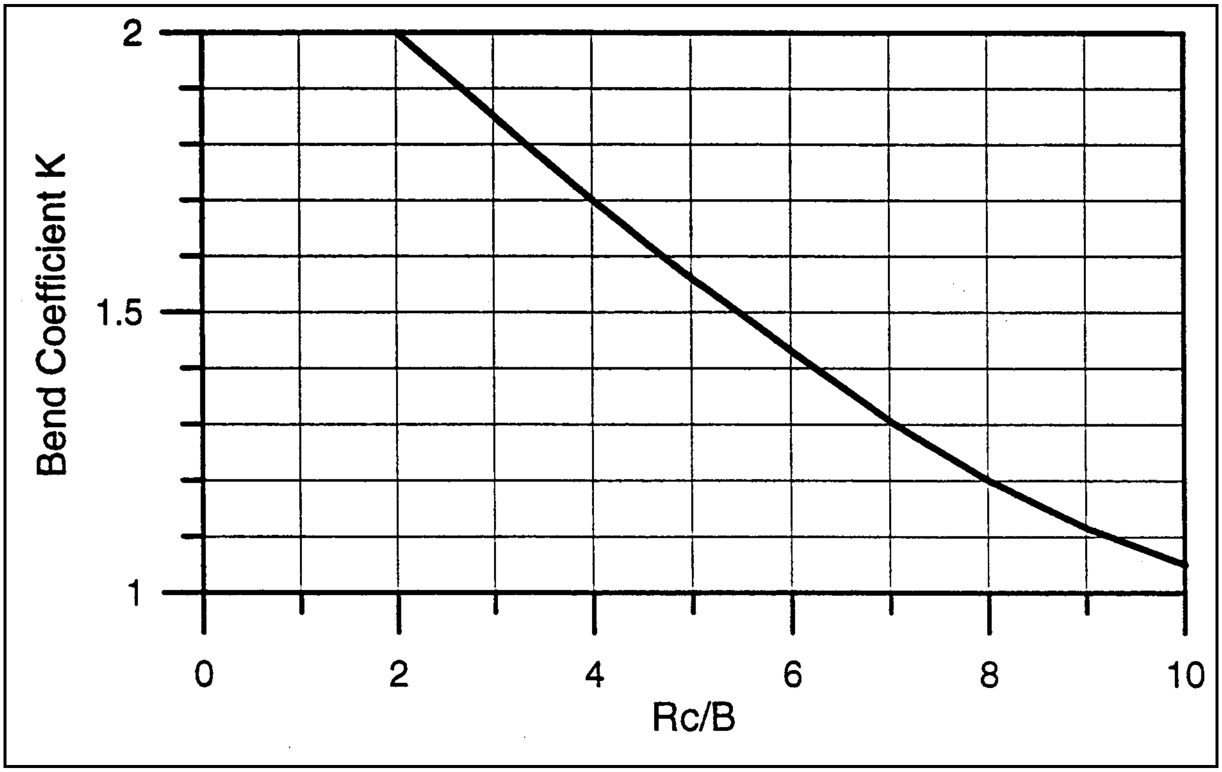


Figure 2.3. Factor for determining shear stress at channel bends.

Step 3

Determine the values for λ and θ from the channel geometry (Figure 2.1):

Channel slope S = 0.05 ft/ft, so $\lambda = \tan^{-1}(0.05) = 2.86$ degrees
Side slope Z = 3H:1V, so $\theta = \tan^{-1}(1/Z) = 18.43$ degrees

Step 4

Determine η using the τ_C values from Table 2.3 for class 45-L block:

$$\eta = \frac{\tau_o}{\tau_c} = \frac{8.20\,\text{lb/ft}^2}{31.7\,\text{lb/ft}^2} = 0.259$$

Step 5

Determine ratio M/N using the fact that the drag force, F_D is assumed equal to the lift force, F_L

$$M/N = \frac{\ell_4}{\ell_3} = \frac{1.22}{0.317} = 3.85$$

Step 6

Determine angle β

$$\beta = \tan^{-1}\left(\frac{\cos\lambda}{\frac{M/N+1}{\eta}\frac{\ell_1}{\ell_2}\sin\theta + \sin\lambda}\right) = \tan^{-1}\left(\frac{\cos(2.86)}{\frac{3.85+1}{0.259}\left(\frac{0.198\text{ft}}{1.22\text{ft}}\right)\sin(18.43) + \sin(2.86)}\right) = 44.66°$$

Step 7

Determine η'

$$\eta' = \eta\left(\frac{\frac{\ell_4}{\ell_3} + \sin(\lambda+\beta)}{\frac{\ell_4}{\ell_3}+1}\right) = 0.259\left(\frac{\frac{1.22}{0.317} + \sin(2.86+44.66)}{\frac{1.22}{0.317}+1}\right) = 0.245$$

Step 8

Determine the additional drag force, F_D', due to a projecting block:

$$F_D' = 0.5\rho\,\Delta z w V^2 = 0.5\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)(0.5\text{in})\left(\frac{1\text{ft}}{12\text{in}}\right)(23.6\text{in})\left(\frac{1\text{ft}}{12\text{in}}\right)\left(13.78\,\frac{\text{ft}}{\text{s}}\right)^2 = 15.1\text{lbs}$$

Step 9

Calculate the factor of safety for class 45-L closed-cell block:

$$SF = \frac{\frac{\ell_2}{\ell_1}\cos\theta}{\sin\theta\cos\beta + \frac{\ell_2}{\ell_1}\eta' + \frac{\ell_3 F_D'}{\ell_1 W_s}\sin(\lambda+\beta) + \frac{\ell_4 F_L''}{\ell_1 W_s}}$$

$$SF = \frac{\frac{1.22}{0.198}\cos(18.43)}{\sin(18.43)\cos(44.66) + \frac{1.22\text{ft}}{0.198\text{ft}}0.245 + \frac{(0.317\text{ft})15.1\text{lbs}}{(0.198\text{ft})56.2\text{lbs}}\sin(2.86+44.66) + \frac{(1.22\text{ft})15.1\text{lbs}}{(0.198\text{ft})56.2\text{lbs}}} = 1.58$$

From the factor of safety analysis, the ArmorFlex® 45-L class blocks exhibit a factor of safety greater than the desired 1.5 even with the assumption that blocks will have a ½-inch projection.

3. REFERENCES

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